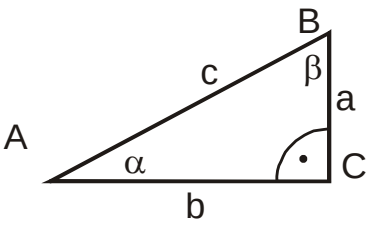
# TRIGONOMETRIJA

Definicije trigonomertijskih funkcija oštrog ugla pravouglog trougla

$$sin\,\alpha = \frac{a}{c}$$
$$cos\,\alpha = \frac{b}{c}$$
$$tg\,\alpha = \frac{a}{b} = \frac{sin\,\alpha}{cos\alpha} = \frac{1}{ctg\alpha}$$




$$sin\,\beta = \frac{b}{c}$$
$$cos\,\beta = \frac{b}{c}$$
$$tg\,\beta = \frac{b}{a} = \frac{sin\,\beta}{cos\beta} = \frac{1}{ctg\beta}$$

$$sin\alpha^2 + cos\alpha^2 = 1$$
$$sin\alpha^2 + cos\alpha^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = 1$$

Jednakostranični trougao

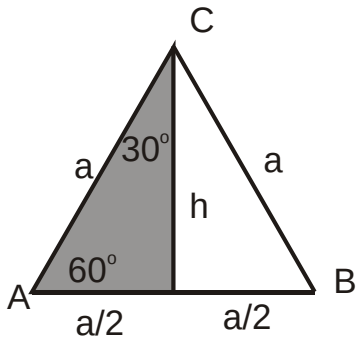
$$sin30° = \frac{\frac{a}{2}}{a} = \frac{a}{2a} = \frac{1}{2}$$

$$sin60° = \frac{h}{a} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$tg\,30° = \frac{\frac{a}{2}}{h} = \frac{\frac{a}{2}}{\frac{a\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$$

$$ctg30° = \frac{h}{a} = \frac{\frac{a\sqrt{3}}{2}}{\frac{a}{2}} = \sqrt{3}$$

$$ctg\,60° = \frac{\frac{a}{2}}{h} = \frac{\frac{a}{2}}{\frac{a\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$$



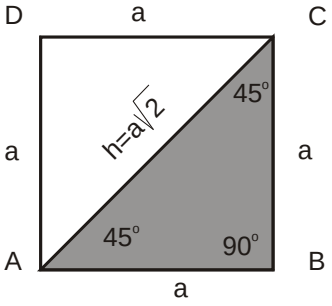

$$cos30° = \frac{h}{a} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$cos60° = \frac{\frac{a}{2}}{a} = \frac{a}{2a} = \frac{1}{2}$$

$$tg60° = \frac{h}{a} = \frac{\frac{a\sqrt{3}}{2}}{\frac{a}{2}} = \sqrt{3}$$

$$h = \frac{a\sqrt{3}}{2}$$

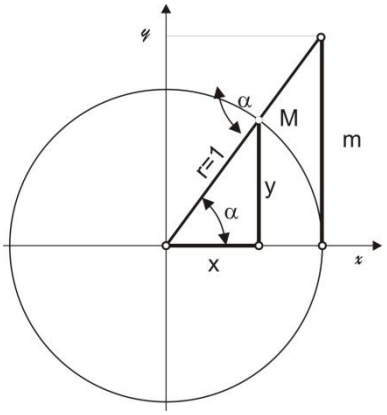
1.2 Jednakokraki trougao sa $\alpha$=45$^o$




$$h = a\sqrt{2}$$
$$sin45° = \frac{a}{d} = \frac{a}{a\sqrt{2}} = \frac{\sqrt{2}}{2} = cos45°$$
$$ctg45° = ctg45° = 1$$

| Funkcija/ugao | 0$^o$ | 30$^o$ = $\frac{\pi}{6}$ | 45$^o$ = $\frac{\pi}{4}$ | 60$^o$ = $\frac{\pi}{3}$ | 90$^o$ = $\frac{\pi}{2}$ |
|---|---|---|---|---|---|
| sin$\alpha$ | 0 | $\frac{1}{2}$=0,5 | $\frac{\sqrt{2}}{2}$=0,707 | $\frac{\sqrt{3}}{2}$=0,866 | 1 |
| cos$\alpha$ | 1 | $\frac{\sqrt{3}}{2}$=0,866 | $\frac{\sqrt{2}}{2}$=0,707 | $\frac{1}{2}$=0,5 | 0 |
| tg$\alpha$ | 0 | $\frac{\sqrt{3}}{3}$=0,577 | 1 | $\sqrt{3}$=1,732 | ∞ |

# Trigonometrijski krug




*r=1*

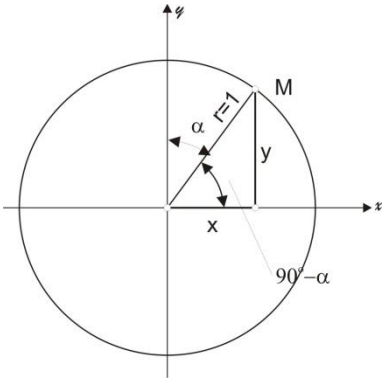
$sin\alpha = \frac{y}{r} = y$

$cos\alpha = \frac{x}{r} = y$

$tg\alpha = \frac{m}{r} = m$

| ugao | 0 | 90 | 180 | 270 | 360 |
|---|---|---|---|---|---|
| sinα | 0 | 1 | 0 | -1 | 0 |
| cosα | 1 | 0 | -1 | 0 | 1 |

Svođenje na oštar ugao




$\sin(90^o-\alpha) = y = \cos\alpha$

$\cos(90-\alpha) = x = \sin\alpha$

***primer:***

$\alpha = 30^o$

$\sin(90^o-\alpha) = \sin(90^o-30^o) = y = \frac{1}{2} = \cos 30^o$

$\cos(90^o-\alpha) = \cos(90^o-30^o) = x = \frac{\sqrt{3}}{2} = \sin 30^o$

$\sin(90^o+\alpha) = y = \cos\alpha$

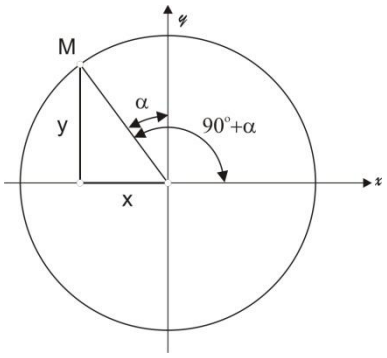
$\cos(90^o+\alpha) = -x = -\sin\alpha$

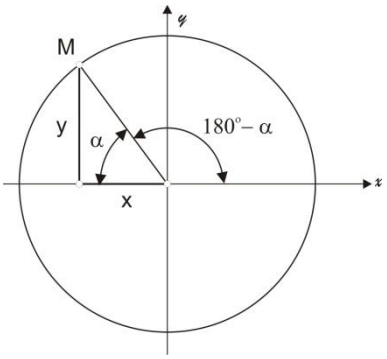
***primer:***

$\alpha = 60^o$

$\sin(90^o+\alpha) = \sin(90^o+60^o) = y = \frac{1}{2} = \cos 60^o$

$\cos(90^o+\alpha) = \cos(90^o+60^o) = - x = - \frac{1}{2} = - \sin 60^o$




$\sin(180^o-\alpha) = y = \sin\alpha$

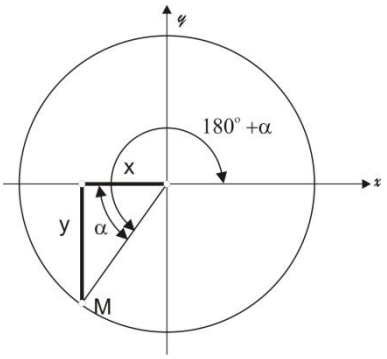
$\cos(180^o-\alpha) = - x = - \cos\alpha$

***primer:***

$\alpha = 30^o$

$\sin(180^o-\alpha) = \sin(180^o-30^o) = y = \frac{1}{2} = \sin 30^o$

$\cos(180^o-\alpha) = \cos(180^o-30^o) = - x = - \frac{\sqrt{3}}{2} = - \cos 30^o$




$\sin(180^o+\alpha) = - y = - \sin\alpha$

$\cos(180^o+\alpha) = - x = - \cos\alpha$

***primer:***

$\alpha = 30^o$

$\sin(180^o+\alpha) = \sin(180^o+30^o) = - y = - \frac{1}{2} = - \sin 30^o$

$\cos(180^o+\alpha) = \cos(180^o+30^o) = - x = - \frac{\sqrt{3}}{2} = - \cos 30^o$

$\sin(360^o-\alpha) = - y = -\sin\alpha$

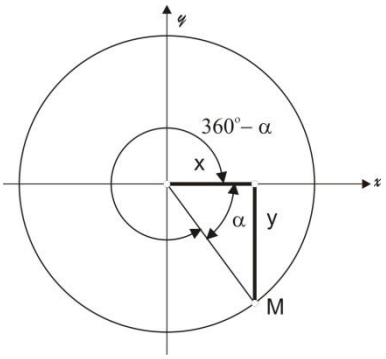
$\cos(360-\alpha) = x = \cos\alpha$

***primer:***

$\alpha = 30^o$

$\sin(360^o-\alpha) = \sin(360^o-30^o) = - y = - \frac{1}{2} = - \sin 30^o$

$\cos(360^o-\alpha) = \cos(360^o-30^o) = x = \frac{\sqrt{3}}{2} = \cos 30^o$



## Adicione formule

Trigonometrijske formule zbira i razlike uglova

$sin(\alpha \pm \beta) = sin\alpha\ cos\beta \pm cos\alpha\ sin\beta$

$cos(\alpha \pm \beta) = cos\alpha\ cos\beta \mp sin\alpha\ sin\beta$

$tg(\alpha \pm \beta) = \frac{tg\alpha \pm tg\beta}{1 \mp tg\alpha\ tg\beta}$

$ctg(\alpha \pm \beta) = \frac{ctg\alpha\ ctg\beta \mp 1}{tg\beta \pm tg\alpha}$

Trigonometrijske funkcije udvojenog ugla

$sin2\alpha = 2\ sin\alpha\ cos\alpha$

primer: $\alpha$=30° $sin30° = \frac{1}{2}$ $cos30° = \frac{\sqrt{3}}{2}\Delta$

$sin2\alpha = 2\frac{1}{2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

2$\alpha$ = 60 ° $sin60° = \frac{\sqrt{3}}{2}$

$cos2\alpha = cos^2\alpha - sin^2\alpha$

primer: $\alpha$=30° $sin30° = \frac{1}{2}$ $cos30° = \frac{\sqrt{3}}{2}$

$cos2\alpha = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

2$\alpha$ = 60 $cos60° = \frac{1}{2}$

Trigonometrijske funkcije polovine ugla

iz relacije

$cos2\alpha = cos^2\alpha - sin^2\alpha$ zamenom $\alpha = 2\frac{\alpha}{2}$

$cos\alpha = cos^2\frac{\alpha}{2} - sin^2\frac{\alpha}{2}$

$sin^2\frac{\alpha}{2} + cos^2\frac{\alpha}{2} = 1$ odavde $sin^2\frac{\alpha}{2} = 1 - cos^2\frac{\alpha}{2}$

$cos\alpha = cos^2\frac{\alpha}{2} - sin^2\frac{\alpha}{2} = cos^2\frac{\alpha}{2} - \left(1 - cos^2\frac{\alpha}{2}\right) = 2cos^2\frac{\alpha}{2} - 1$

$2cos^2\frac{\alpha}{2} = cos\alpha + 1$ ⟹ $\boxed{cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+cos\alpha}{2}}}$

$cos2\alpha = cos^2\alpha - sin^2\alpha$ zamenom $\alpha = 2\frac{\alpha}{2}$

$cos\alpha = cos^2\frac{\alpha}{2} - sin^2\frac{\alpha}{2}$

$sin^2\frac{\alpha}{2} + cos^2\frac{\alpha}{2} = 1$ odavde

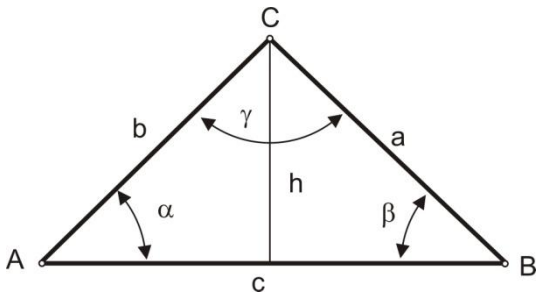
$cos^2\frac{\alpha}{2} = 1 - sin^2\frac{\alpha}{2}$

$cos\alpha = cos^2\frac{\alpha}{2} - sin^2\frac{\alpha}{2} = 1 - sin^2\frac{\alpha}{2} - - sin^2\frac{\alpha}{2} = 1 - 2sin^2\frac{\alpha}{2}$

$sin^2\frac{\alpha}{2} = \frac{1-cos\alpha}{2}$ ⇒ $\boxed{sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-cos\alpha}{2}}}$

$sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-cos\alpha}{2}}$ $cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+cos\alpha}{2}}$

## Sinusna teorema




$\sin\alpha = \frac{h}{b}$ $\quad h = b \sin\alpha$

$\sin\beta = \frac{h}{a}$ $\quad h = a \sin\beta$

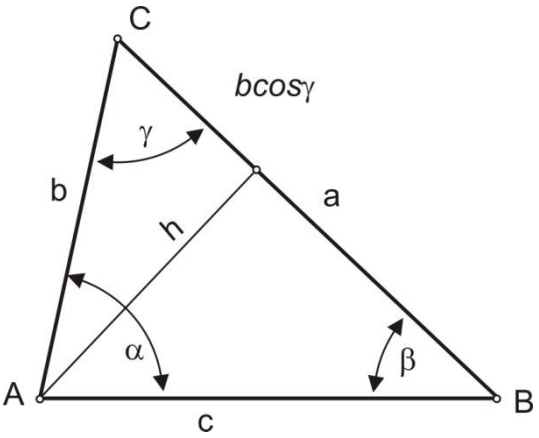
$$\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$

## Cosinusna teorema

$h = b \sin\gamma$

$c^2 = h^2 + (a - b\cos\gamma)^2 =$

$= b^2\sin^2\gamma + a^2 - 2ab\cos\gamma + b^2\cos^2\gamma =$

$= a^2 + b^2(\sin^2\gamma + \cos^2\gamma) - 2ab\cos\gamma$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$