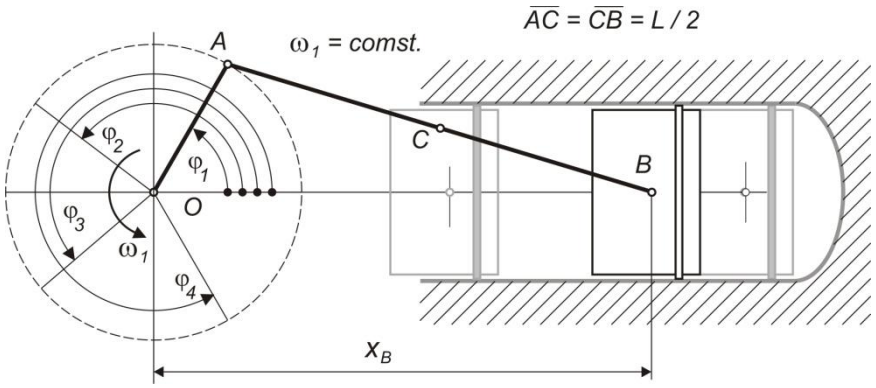
Zadatak:

Klipni mehanizam se sastoji iz krivaje (ekscentarske poluge) OA dužine R, klipne poluge AB dužine L=3R i klipa kompresora B (ukrsne glave). Krivaja se okreće konstantnim brojem obrtaja n.

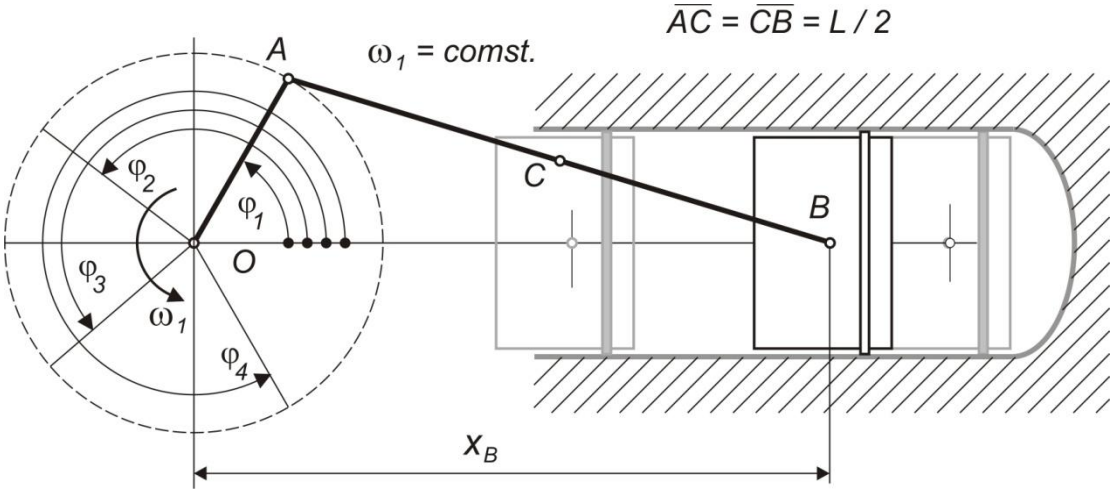


Podaci za proračun						
n o/min	R m	L m	φ_1 °	φ_2 °	φ_3 °	φ_4 °
20	0.2	0.6	30	135	210	300

Za date vrednosti odrediti:

a) analitičkim putem položaj klipa za $\varphi 1$, $\varphi 2$, $\varphi 3$, $\varphi 4$, dati izraz i tabelu rezultata
b) analitičkim putem brzinu i ubrzanje klipa B za ugao krivaje $\varphi 1$
c) vektorskom metodom brzinu i ubrzanje klipa B za ugao krivaje φ 2
d) grafičkim metodom zaokrenutih brzina odrediti brzinu klipa B za ugao krivaje $\varphi 1$
e) metodom plana brzina i plana ubrzanja odrediti brzine klipa B za ugao krivaje $\varphi 2$
f) metodom plana ubrzanja odrediti ubrzanje tačake C za ugao krivaje $\varphi 1$

a) ANALITIČKI NAČIN ODREĐIVANJA POLOŽAJA KLIPA



Kako je data konstantna tehnička ugaona brzina – broj obrtaja u minuti određuje se ugaona brzina

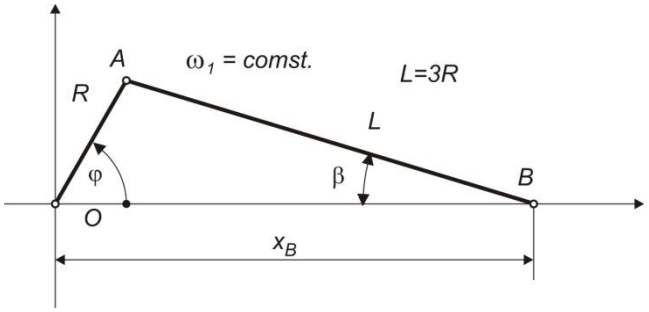
$\omega_1 = \frac{\pi \cdot n}{30} = \frac{\pi \cdot 20}{30} = 2.09 \frac{1}{s} = const.$

Zakon promene ugla poluge

$\varphi = \int \omega_1 dt + C_1 = \omega_1 \int dt + C_1 = \omega_0 t$

Pošto se ugao meri od 0 kao što je prikazano na slici C_1=0

$\varphi = \omega_1 t = 2.094\, t$ rad



$\dot{\varphi} = \omega_1 = 2.094 \ \frac{1}{s}$

$\ddot{\varphi} = 0$

Zakon kretanja tačke A

$x_A = Rcos\varphi = 0.2cos\varphi$ m

$y_A = Rsin\varphi = 0.2sin\varphi$ m

Zakon kretanja tačke B

$x_B = Rcos\varphi + Lcos\beta$ $\qquad$ $y_B = 0$

Primenom sinusne teoreme na trougao OAB određuje se

$\frac{R}{sin\beta} = \frac{L}{sin\varphi} = \frac{x_B}{sin(180° - \beta - \varphi)}$

$sin\beta = \frac{R}{L} sin\varphi = \lambda sin\varphi$ gde je $\lambda = \frac{R}{L} = \frac{1}{3}$

$cos\beta = \sqrt{1 - sin^2\beta} = \sqrt{1 - \lambda^2 sin^2\varphi}$

$x_B = Rcos\varphi + Lcos\beta = Rcos\varphi + L\sqrt{1 - \lambda^2 sin^2\varphi}$

Radi jednostavnijeg daljeg rada izraz za cosβ se razvija u red i uzimaju samo prva dva člana pa izraz za x_B glasi

$$cos\beta = \sqrt{1-\lambda^2 sin^2\varphi} \cong 1 - \frac{1}{2}\lambda^2 sin^2\varphi + \cdots$$

$$x_B = Rcos\varphi + Lcos\beta = Rcos\varphi + L\left(1 - \frac{1}{2}\lambda^2 sin^2\varphi\right) = R\left(cos\varphi - \frac{1}{2}\lambda sin^2\varphi\right) + L$$

$$x_B = R\left(cos\varphi + \frac{1}{4}\lambda cos2\varphi\right) + \left(L - \frac{1}{4}\lambda R\right)$$

Za brojne vrednosti iz zadatka

$$x_B = 0.2\left(cos\varphi + \frac{1}{4}\cdot\frac{1}{3}cos2\varphi\right) + \left(0.6 + \frac{1}{4}\cdot\frac{1}{3}0.2\right) = 0.2\left(cos\varphi + \frac{1}{12}cos2\varphi\right) + 0.58333$$

φ	$cos\varphi$	$cos2\varphi$	x_B bez aproksimacije m	x_B m
30	0.866025403	0.5	0.764813059	0.76487171
135	-0.7070106781	0	0.441673832	0.44191197
210	-0.866025403	0.5	0.418402897	0.41846155
300	0.5	-0.5	0.674456264	0.64999966

b) ANALITIČKI NAČIN ODREĐIVANJA BRZINE I UBRZANJA

$$x_B = R\left(cos\varphi + \frac{1}{4}\lambda cos2\varphi\right) + \left(L - \frac{1}{4}\lambda R\right)$$

$$V_B = \dot{x}_B = -R\dot{\varphi}\left(sin\varphi + \frac{1}{2}\lambda sin2\varphi\right)$$

$$a_B = \ddot{x}_B = R\dot{\varphi}^2(cos\varphi + \lambda cos2\varphi)$$

Za ugao $\varphi = 30° = \frac{\pi}{6}$ rad p sledi

$$\dot{\varphi} = \omega_1 = 2.094 \ = const. \ \frac{1}{s}$$

$$\ddot{\varphi} = 0$$

$$x_B = 0.2\left(cos\frac{\pi}{6} + \frac{1}{4}\lambda cos2\frac{\pi}{6}\right) + \left(0.6 - \frac{1}{4}\cdot\frac{1}{3}0.2\right) = \text{ 0.764813059 m}$$

$$V_B = \dot{x}_B = -0.2\cdot 2.091\ \left(sin\frac{\pi}{6} + \frac{1}{2}\cdot\frac{1}{3}sin2\frac{\pi}{6}\right) \text{=-0.26946197 m/s}$$

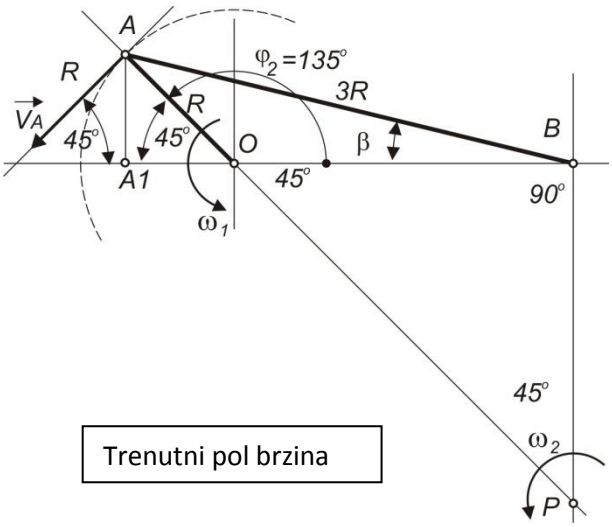
$$a_B = \ddot{x}_B = 0.2\cdot 2.091^{\,2}\left(cos\frac{\pi}{6} + \frac{1}{3}cos\frac{\pi}{3}\right) \text{=0.90304 m/s}^2$$

c) VEKTORSKI NAČIN ODREĐIVANJA BRZINE I UBRZANJA

$\vec{V_B} = \overrightarrow{V_A} + \overrightarrow{V_{BA}}$ Za ugao $\varphi = 135° = \frac{3\pi}{4}$ rad

C1 - primenom trenutnog pola brzina

$$V_A = R\cdot\omega_1 = R\dot{\varphi} = 0.2\cdot 2.094 = 0.4188\ m/s$$



Trenutni pol brzina

$$sin\varphi = sin135° = \frac{\sqrt{2}}{2}$$

$\frac{R}{sin\beta} = \frac{L}{sin\varphi}$ odakle se izračunava

$$sin\beta = \frac{sin\varphi R}{3R} = \frac{\sqrt{2}}{6}$$

$$\cos\beta = \sqrt{1 - sin^2\beta} = \sqrt{\frac{36-2}{36}} = \frac{\sqrt{34}}{6}$$

$$\overline{BO} = \overline{AA_1} - R\frac{\sqrt{2}}{2} = Lcos\beta - R\frac{\sqrt{2}}{2}$$

$$\overline{BO} = 3Rcos\beta - R\frac{\sqrt{2}}{2} = R\left(3\frac{\sqrt{34}}{6} - \frac{\sqrt{2}}{2}\right)$$

$$\overline{BO} = \frac{R}{2}\left(\sqrt{34} - \sqrt{2}\right) = 2.208369\,R$$

$$\overline{BO} = 0.4416738\,m$$

$$\overline{AO} = R,$$

pošto je trougao OBP jednakokreki pravougli sa uglovima 45°90°45° to je

$$\overline{BO} = \overline{PB} = 2.208369\,R = 0.4416738\,m$$

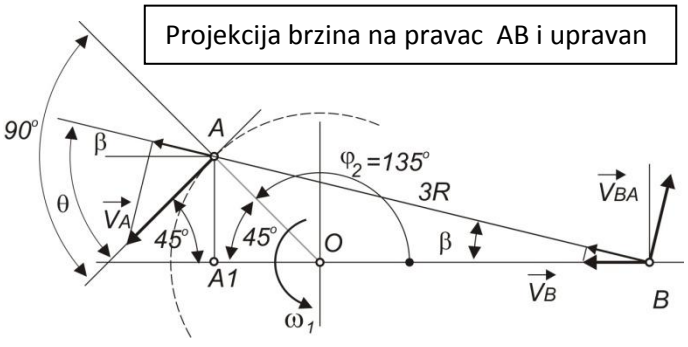
$$\overline{OP} = \overline{BO}\sqrt{2} = 2.208369\ \cdot\sqrt{2}\,R = 3.123105R$$

$$\overline{AP} = \overline{BO}\sqrt{2} + R = 3.123105R\ + R = 4.12305\,R = 0.824621\,m$$

$$\omega_2 = \frac{V_A}{\overline{AP}} = \frac{0.4188}{0.824621} = 0.507869\ \ 1/s$$

$$V_B = \omega_2 \cdot \overline{PB} = 0.540513\ \cdot 0.4416738 = 0.2243127\,m/s$$

C2 - Projekcija brzina na pravac koji spaja tačke čije se brzine traže



$$\vec{V}_B = \vec{V_A} + \vec{V_{BA}}$$

Sa slike se vidi da je ugao brzine V_A sa pravcem AB θ jednak:

$$\theta = 45° + \beta$$

$$sin\beta = \frac{sin\varphi R}{3R} = \frac{\sqrt{2}}{6}$$

$$\beta = arcsin\frac{\sqrt{2}}{6} = 13.633°$$

$$\theta = 45° + \beta = 45° + 13.633° = 58.633°$$

Projekcija brzina na pravac AB

$$V_A cos\theta = V_B cos\beta$$

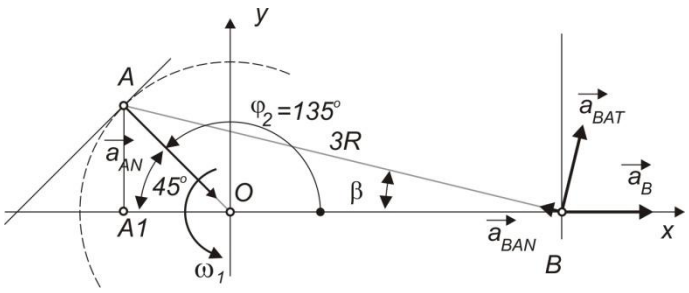
$V_B = \frac{V_A cos\theta}{cos\beta} = \frac{0.4188 \cdot cos 58.633°}{\cos 13.633°} = 0.2243\ m/s$

Projekcija na pravac upravan na AB

$V_{BA} = V_A sin\theta - V_B sin\beta = 0.4188 \cdot sin58.633° - 0.2243\ sin13.633 = 0.30472\ m/s$

Ugaona brzina člana AB

$\omega_2 = \frac{V_{BA}}{L} = \frac{0.3047}{0.6} = 0.50787\ \ 1/s$



Ubrzanja tačaka su

$\vec{a}_B = \overrightarrow{a_A} + \vec{a}_{BA\varepsilon} + \vec{a}_{BA\omega}$

$\vec{a}_A = \vec{a}_{A\varepsilon} + \vec{a}_{A\omega}$

Kako je konstantna ugaona brzina ubrzanje tačke A od ugaonog ubrzanja je jednako nuli i ubrzanje je jednako ubrzanju kao posledice ugaone brzine

$\vec{a}_{A\varepsilon} = 0$

$a_{A\omega} = R\ {\omega_1}^2 = 0.2 \cdot 2.094^2 = 0.87729\ m/s^2$

$\vec{a}_A = \vec{a}_{A\varepsilon} + \vec{a}_{A\omega} = 0 + \vec{a}_{A\omega} = \vec{a}_{A\omega}$

$a_{BA\omega} = L\ {\omega_2}^2 = 0.6 \cdot 0.50787^2 = 0.15475\ m/s^2$

Projekcije jedne i druge strane izraza na x i y osu

$\vec{a}_B = \overrightarrow{a_A} + \vec{a}_{BA\varepsilon} + \vec{a}_{BA\omega}$

X: $a_B = a_A \cos 45° - a_{BA\omega} cos\beta + a_{BA\varepsilon} sin\beta$

y: $0 = -a_A \sin 45° + a_{BA\omega} sin\beta + a_{BA\varepsilon} cos\beta$ odavde se dobija $a_{BA\varepsilon}$

$a_{BA\varepsilon} = \frac{a_A \sin 45° - a_{BA\omega}\ sin\beta}{cos\beta} = \frac{a_A \cdot \sin 45° - a_{BA\omega} \cdot sin\ 13.633°}{cos\ 13.633}$

$a_{BA\varepsilon} = \frac{0.87729 \cdot \sin 45° - 0.15475 \cdot sin\ 13.633°}{cos 13.633°} = \frac{0.87729 \cdot 0.7071 - 0.15475 \cdot 0.235701}{0.971825}$

$a_{BA\varepsilon} = 0.6007898\ m/s^2$ zmenom u izraz projekcija na x osu

$a_B = a_A \cos 45° - a_{BA\omega} cos\beta + a_{BA\varepsilon} sin\beta$

$a_B = a_A \cos 45° - a_{BA\omega} cos 13.633° + a_{BA\varepsilon} sin 13.633°$

$a_B = 0.87729 \cdot 0.7071 - 0.15475 \cdot 0.971825 + 0.6007898 \cdot 0.235701$

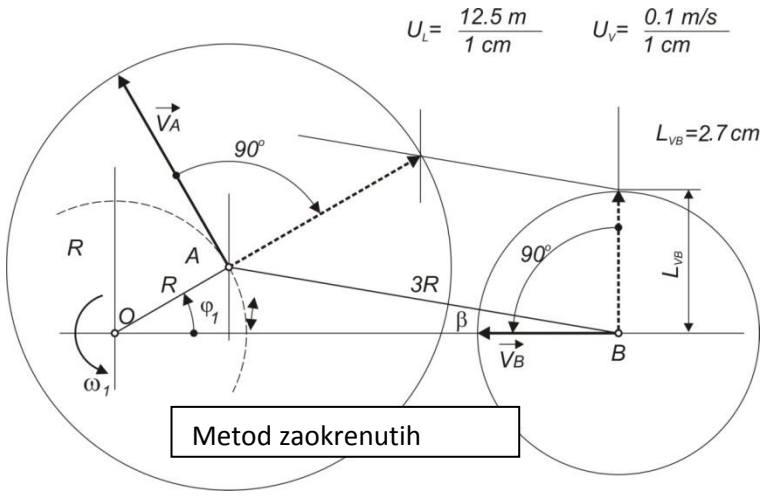
$a_B = 0.6115\ m/s^2$

d) GRAFIČKI METOD ZAOKRENUTIH BRZINA

Za nacrtati krivajni mehanizam u odabranoj razmeri

Izračunati brzinu tačke A V_A i nacrtati je u odabranoj razmeri na crtežu mehanizma

$$V_A = R \cdot \omega_1 = R\dot{\varphi} = 0.2 \cdot 2.094 = 0.4188\ m/s$$



$U_V = \frac{0.1\ m/s}{1\ cm}$ pa se izračunava $L_{VA} = \frac{V_A}{U_V} = \frac{0.4188\ m/s}{\frac{0.1\ m/s}{1\ cm}} = 4.18\ cm$

Grafički u tački B nacrtati pravac normalan na pravac brzine V_B

Zaokrene se V_A za 90° oko tačke Ai kroz kraj povuče paralelna linija sa AB

U preseku sa upravnim pravcem na brzinu V_B kroz tačku B dobija se vrednost te brzine i zaokretanjem za 90° dobija se tražena brzina

izračunava $V_{\boldsymbol{B}} = L_{VB} \cdot U_V = 2.7cm\ \frac{0.1\ m/s}{1\ cm} = 0.27\ m/s$

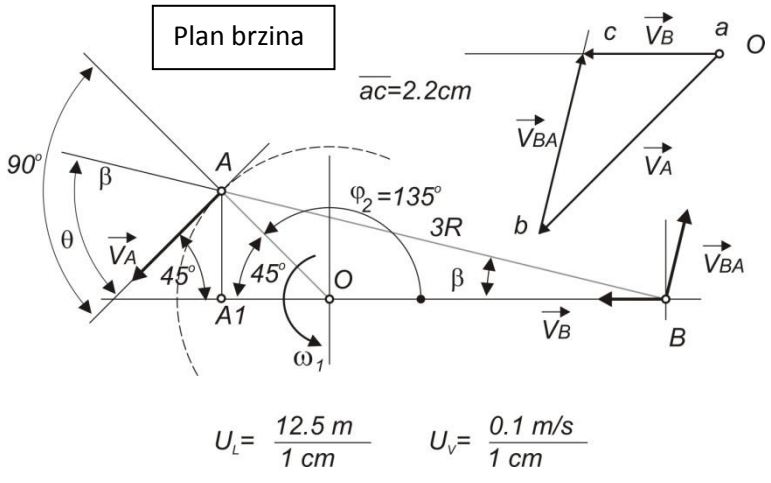
e) METOD PLANA BRZINA I PLANA UBRZANJA

Na osnovu zadate ugaone brzine izračunaju se

$$V_A = R \cdot \omega_1 = R\dot{\varphi} = 0.2 \cdot 2.094 = 0.4188\ m/s$$

$U_V = \frac{0.1\ m/s}{1\ cm}$ pa se izračunava $\overline{ab} = \frac{V_A}{U_V} = \frac{0.4188\ m/s}{\frac{0.1\ m/s}{1\ cm}} = 4.18\ cm$

$$\vec{a}_A = \vec{a}_{A\varepsilon} + \vec{a}_{A\omega} = 0 + \vec{a}_{A\omega} = \vec{a}_{A\omega} = R \cdot \omega^2{}_1 = 0.87729\,\frac{m}{s^2}$$



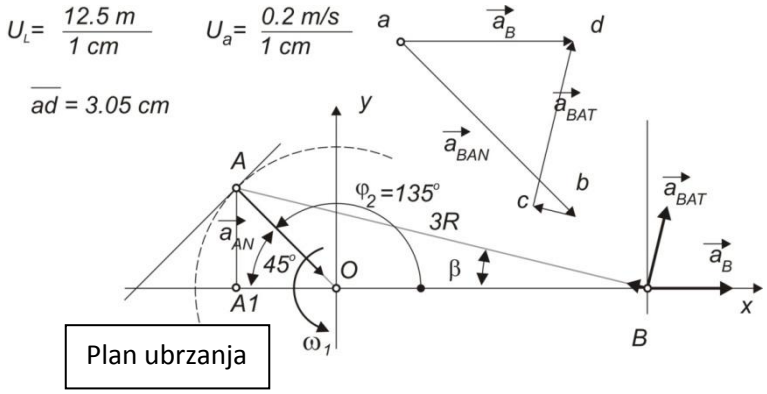
Na osnovu izmerene veličine sa slika plana brzina izračunava

$V_{\boldsymbol{B}} = \overline{ac} \cdot U_V = 2.2cm\ \frac{0.1\frac{m}{s}}{1\ cm}$

$V_{\boldsymbol{B}} = 0.22\frac{m}{s}$

U_L= 12.5 m / 1 cm U_V= 0.1 m/s / 1 cm

$U_a = \frac{0.2\ m/s^2}{1\ cm}$ pa se izračunava $\overline{ab} = \frac{a_A}{U_a} = \frac{0.87729\frac{m}{s}}{\frac{0.2\frac{m}{s}}{1\ cm}} = 4.38\ cm$



Plan ubrzanja

$a_{\boldsymbol{B}} = \overline{ad} \cdot U_a = 3.05cm\ \frac{0.2\frac{m}{s^2}}{1\ cm}$

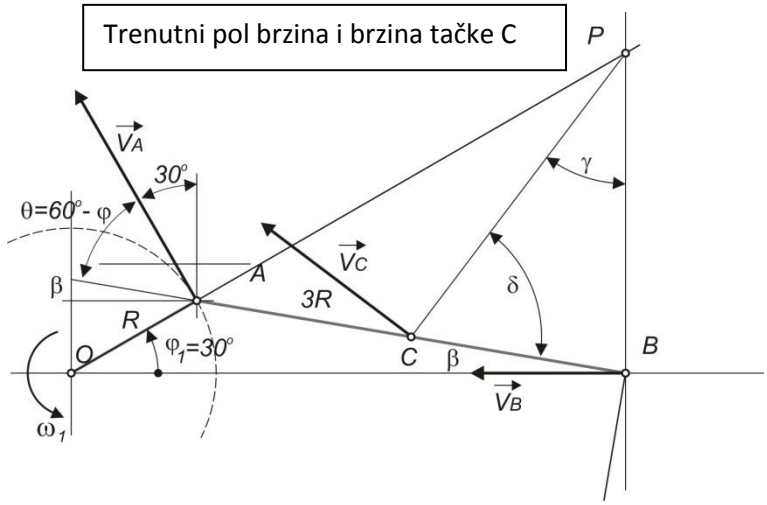
$a_{\boldsymbol{B}} = 0.61\frac{m}{s^2}$

f) METOD PLANA PLANA UBRZANJA UBRZANJE TAČKE C

Da bi se odredilo ubrzanje tačke C treba odrediti ugaonu brzinu i ugaono ubrzanje poluge AB

$\vec{V}_B = \overrightarrow{V_A} + \overrightarrow{V_{BA}}$

Trenutni pol brzina i brzina tačke C



Sa slike se vidi da je ugao brzine V_A sa pravcem AB θ jednak:

$\theta = 60° - \beta$

$sin\beta = \frac{sin\varphi R}{3R} = \frac{0.5}{3}$

$\beta = arcsin\frac{1}{6} = 9.594°$

$\theta = 60° - \beta = 60° - 9.594° = 50.405°$

Projekcija brzina na pravac AB

$V_A cos\theta = V_B cos\beta$

$V_B = \frac{V_A cos\theta}{cos\beta} = \frac{0.4188 \cdot cos 50.405°}{\cos 9.594°} = 0.2707\ m/s$

Projekcija na pravac upravan na AB

$V_{BA} = V_A sin\theta - V_B sin\beta = 0.4188 \cdot sin 50.405° - 0.2707\ sin 9.594 = 0.3678\ m/s$

Ugaona brzina člana AB

$\omega_2 = \frac{V_{BA}}{L} = \frac{0.36784}{0.6} = 0.61306\ 1/s$

Ubrzanja tačaka su

$\vec{a}_B = \overrightarrow{a_A} + \vec{a}_{BA\varepsilon} + \vec{a}_{BA\omega}$

$\vec{a}_A = \vec{a}_{A\varepsilon} + \vec{a}_{A\omega}$

Kako je konstantna ugaona brzina ubrzanje tačke A od ugaonog ubrzanja je jednako nuli i ubrzanje je jednako ubrzanju kao posledice ugaone brzine

$\vec{a}_{A\varepsilon} = 0$

$a_{A\omega} = R\ {\omega_1}^2 = 0.2 \cdot 2.094^2 = 0.87729\ m/s^2$

$\vec{a}_A = \vec{a}_{A\varepsilon} + \vec{a}_{A\omega} = 0 + \vec{a}_{A\omega} = \vec{a}_{A\omega}$

$a_{BA\omega} = L\ {\omega_2}^2 = 0.6 \cdot 0.61306^{\,2} = 0.225508/s^2$

$a_{CA\omega} = \frac{L}{2}\ {\omega_2}^2 = 0.3 \cdot 0.61306^{\,2} = 0.11295\ m/s^2$

Projekcije jedne i druge strane izraza na x i y osu

$\vec{a}_B = \overrightarrow{a_A} + \vec{a}_{BA\varepsilon} + \vec{a}_{BA\omega}$

X: $a_B = a_A \cos 30° - a_{BA\omega} cos\beta + a_{BA\varepsilon} sin\beta$

y: $0 = -a_A \sin 30° + a_{BA\omega} sin\beta + a_{BA\varepsilon} cos\beta$ odavde se dobija $a_{BA\varepsilon}$

$a_{BA\varepsilon} = \frac{a_A \sin 30° - a_{BA\omega}\ sin\beta}{cos\beta} = \frac{a_A \cdot \sin 30° - a_{BA\omega} \cdot sin 9.594°}{cos 9.594}$

$a_{BA\varepsilon} = \frac{0.87729 \cdot \sin 30° - 0.22508 \cdot sin 9.594°}{cos 9.594°} = \frac{0.87729 \cdot 0.7071 - 0.22508 \cdot 0.16666}{0.986013} = 0.40113\ \mathrm{m/s^2}$

$a_{BA\varepsilon} = 0.40133\ m/s^2$ zmenom u izraz projekcija na x osu

$a_B = a_A \cos 30° - a_{BA\omega} cos\beta + a_{BA\varepsilon} sin\beta$

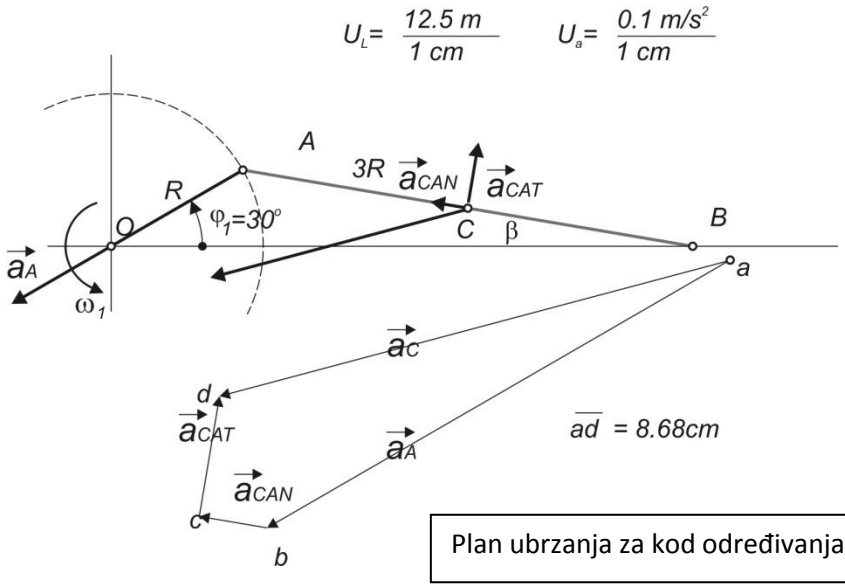
$a_B = a_A \cos 30° - a_{BA\omega} cos 9.594° + a_{BA\varepsilon} sin 9.594°$

$a_B = 0.87729 \cdot 0.86602 - 0.225508 \cdot 0.986013 + 0.40133 \cdot 0.16666$

$a_B = 0.78903\ m/s^2$

$\varepsilon_{AB} = \frac{a_{BA\varepsilon}}{L} = \frac{0.40133}{0.6} = 0.6686\ 1/s^2$

$a_{CA\varepsilon} = \frac{L}{2}\varepsilon_{AB} = 0.3 \cdot 0.6685 = 2.005\ \mathrm{m/s^2}$



Plan ubrzanja za kod određivanja ubrzanja tačke C

$a_c = \overline{ad} \cdot U_a = 8.68cm\ \frac{0.1\frac{m}{s^2}}{1\ cm} = 0.868\frac{m}{s^2}$