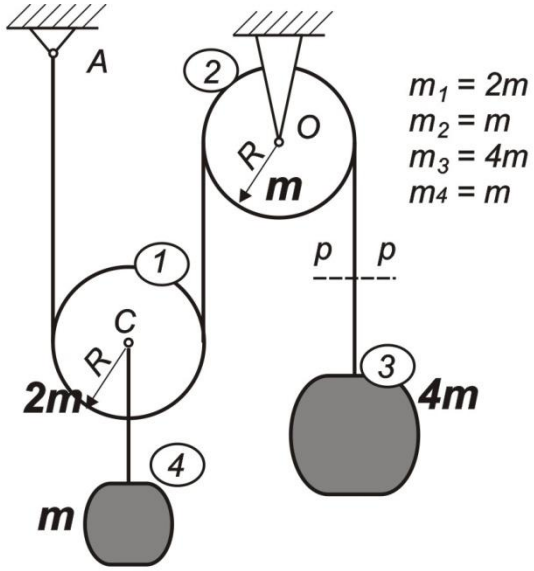
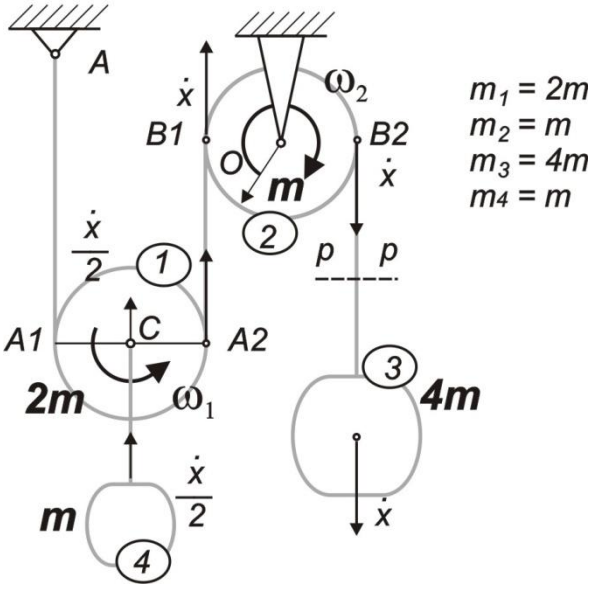




Zadatak 7. Materijalni sistem prikazan na slici sastoji se od kotura 1 mase 2m, kotura 2 mase m. I tereta 3 i 4. Za nepokretnu tačku A vezano je nerastegljivo idealno uže i prebačeno prko kalemova. Na kraj užeta okačen je teret 3 mase 4m. Za center kotura 1 vezan je teret 4 mase m. Kotur se slobobodno obrće oko centra C i pomera pod dejstvom sila težine tegova. Kotur 2 se slobodno obrće oko nepokretnog centra O.

Za dati raspored masa uočava se da će se teret 4 kretati naniže.Kotur 2 se obrće oko nepomičnog centra pa su brzine tačaka B1 i B2 jednake suprotnih smerova. Preko kotura 2 prebačeno nerastegljivo uže. Kraj vezan za nepomičnu tačku A uslovljava da se kotur kotrlja pa je trenutni pol obrtanja tačka A1. Njena brzina je 0 a brzina tačke A2 jednaka brzini tačke B1.

Brzina i ubrzanje težišta tela 1

$$V_1 = \frac{\dot{x}}{2} \quad a_1 = \frac{\ddot{x}}{2}$$

Ugaona brzina i ugaono ubrzanje tela 1

$$\omega_1 = \frac{V_1}{2R} = \frac{\dot{x}}{2R} = \dot{\varphi}_1 \quad \varepsilon_1 = \frac{\ddot{x}}{2R} = \ddot{\varphi}_1$$

Ugaona brzina i ugaono ubrzanje tela 2

$$\omega_2 = \frac{V_1}{R} = \frac{2\dot{x}}{R} = 2\dot{\varphi}_1$$

$$\varepsilon_2 = \frac{2\ddot{x}}{R} = 2\ddot{\varphi}_1$$

$$J_{o1} = \frac{1}{2}2mR^2 = mR^2$$

$$J_{o2} = \frac{1}{2}mR^2$$

Brzina i ubrzanje težišta tela 3

$$V_3 = 2V_1 = \dot{x} \quad a_3 = 2a_1 = \ddot{x}$$

Brzina i ubrzanje težišta tela 4

$$V_4 = V_1 = \frac{\dot{x}}{2} \quad a_4 = a_1 = \frac{\ddot{x}}{2}$$

Kinetička energija sistema

$$E_K = E_k^{(1)} + E_k^{(2)} + E_k^{(3)} + E_k^{(4)}$$

$$E_k^{(1)} = E_{kROT}^{(1)} + E_{kTR}^{(1)}$$

$E_{kROT}^{(1)} = \frac{1}{2}J_{o1}\omega_1^2 = \frac{1}{2}mR^2\left(\frac{\dot{x}}{2R}\right)^2 = \frac{1}{8}m\dot{x}^2$

$E_{kTR}^{(1)} = \frac{1}{2}2m\left(\frac{\dot{x}}{2}\right)^2 = \frac{1}{4}m\dot{x}^2$

$E_k^{(1)} = E_{kROT}^{(1)} + E_{kTR}^{(1)} = \frac{1}{8}m\dot{x}^2 + \frac{1}{4}m\dot{x}^2 = \frac{3}{8}m\dot{x}^2$

$E_k^{(2)} = E_{kROT}^{(2)} = \frac{1}{2}J_{o2}\omega_2^2 = \frac{1}{2}\frac{1}{2}mR^2\left(\frac{\dot{x}}{R}\right)^2 = \frac{1}{4}m\dot{x}^2$

$E_{kTR}^{(3)} = \frac{1}{2}4m(\dot{x})^2 = 2m\dot{x}^2$

$E_{kTR}^{(4)} = \frac{1}{2}m\left(\frac{\dot{x}}{2}\right)^2 = \frac{1}{8}m\dot{x}^2$

$E_K = E_k^{(1)} + E_k^{(2)} + E_k^{(3)} + E_k^{(4)} = \frac{3}{8}m\dot{x}^2 + \frac{1}{4}m\dot{x}^2 + 2m\dot{x}^2 + \frac{1}{8}m\dot{x}^2 = \frac{22}{8}m\dot{x}^2$

Rad sistema

$A = A^{(1)} + A^{(3)} + A^{(4)} = -2mg\frac{x}{2} + 4mgx - mg\frac{x}{2} = \frac{5}{2}mgx$

Promena kinetičke energije jednaka je promeni rada

$\frac{dE_K}{dt} = \frac{dA}{dt}$

$\frac{22}{8}m2\dot{x}\cdot\ddot{x} = \frac{5}{2}mg\dot{x} \rightarrow \ddot{x} = \frac{5}{11}g$

Ravnoteža mase 3 kako bi se odredila sila u užetu

$-4m\ddot{x} = S - 4mg \rightarrow S = 4mg - 4m\ddot{x} = m\left(4g - 4\cdot\frac{5}{11}g\right) = \frac{44-20}{11}mg = \frac{24}{11}mg$

$S = \frac{24}{11}mg$

Rešenje primenom Dalamberovog pricipa uz predpostvka da je brzina tereta 3

$V_3 = 2V_1 = 2\dot{x}$

Brzina i ubrzanje težišta tela 1

$V_1 = \dot{x} \quad a_1 = \ddot{x}$

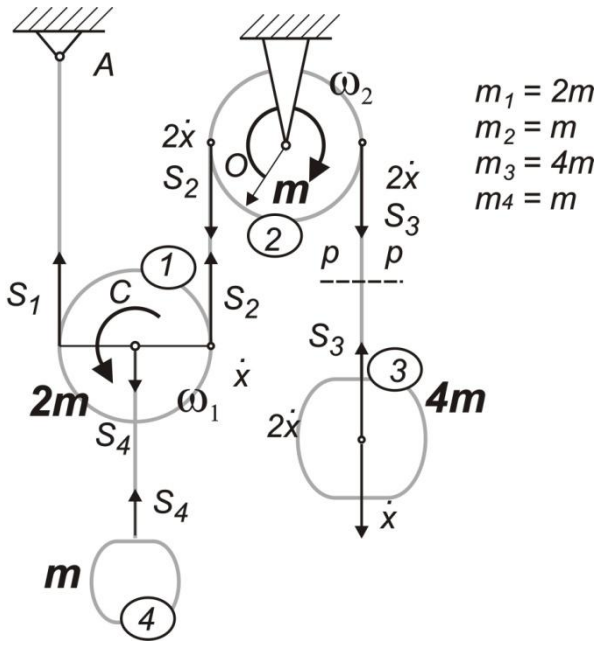
Ugaona brzinai ubrzanje tela 1

$\omega_1 = \frac{V_1}{R} = \frac{\dot{x}}{R} = \dot{\varphi}_1 \quad \varepsilon_1 = \frac{\ddot{x}}{R} = \ddot{\varphi}_1$

Brzina i ubrzanje težišta tela 2

$\boldsymbol{\omega_2} = \frac{2\dot{x}}{R} \quad \ddot{\varphi}_2 = \frac{2\ddot{x}}{R}$

Ugaona brzinai ubrzanje tela 3 $V_3 = 2V_1 = 2\dot{x} \quad a_3 = 2a_1 = 2\ddot{x}$



Zakoni kretanja

1) $ma_1 = S_4 - mg$
2) $J_{o1}.\ddot{\varphi}_1 = S_1\cdot R - S_2\cdot R$

3) $2m\ddot{x} = S_1 + S_2 - 2mg - S_4$
4) $-4ma_2 = S_3 - 4mg$
5) $-J_{o2}.\ddot{\varphi}_2 = S_2 \cdot R - S_3 \cdot R$

$J_{o1} = \frac{1}{2}2mR^2 = mR^2 \quad J_{o2} = \frac{1}{2}mR^2$

Zamenom vrednosti dobija se

1) $m\ddot{x} = S_4 - mg$
2) $mR^2 \frac{\ddot{x}}{R} = S_1 \cdot R - S_2 \cdot R$
3) $-2m\ddot{x} = S_1 + S_2 - 2mg - S_4$
4) $-4m\ddot{x} = S_3 - 4mg$
5) $-\frac{1}{2}mR^2 \frac{2\ddot{x}}{R} = S_2 \cdot R - S_3 \cdot R$

$S_4 = mg - m\ddot{x}$

$S_3 = 4mg - 4m\ddot{x}$

$S_2 = S_3 - m\ddot{x} = 4mg - 5m\ddot{x}$

$S_1 = S_2 + m \cdot \ddot{x} = 4mg - 4m\ddot{x}$

$-2m\ddot{x} = 4mg - 4m\ddot{x} + 4mg - 5m\ddot{x} - 2mg - mg + m\ddot{x}$

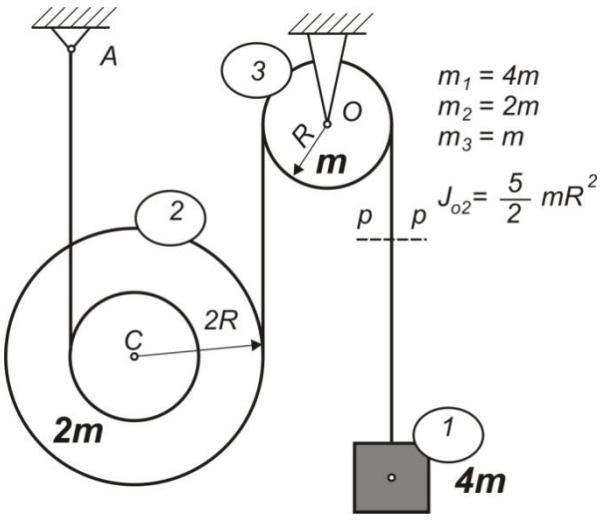
$\ddot{x} = \frac{5}{11}g$

$S_1 = 4mg - 4m\ddot{x} = 4mg - 4m\frac{5}{11}g = \frac{24}{11}mg$

$S_2 = 4mg - 5m\ddot{x} = 4mg - 5m\frac{5}{11}g = \frac{19}{11}mg$

$S_3 = 4mg - 4m\ddot{x} = 4mg - 4m\frac{5}{11}g = \frac{24}{11}mg$

$S_4 = mg - m\ddot{x} = mg - m\frac{5}{11}g = \frac{6}{11}mg$

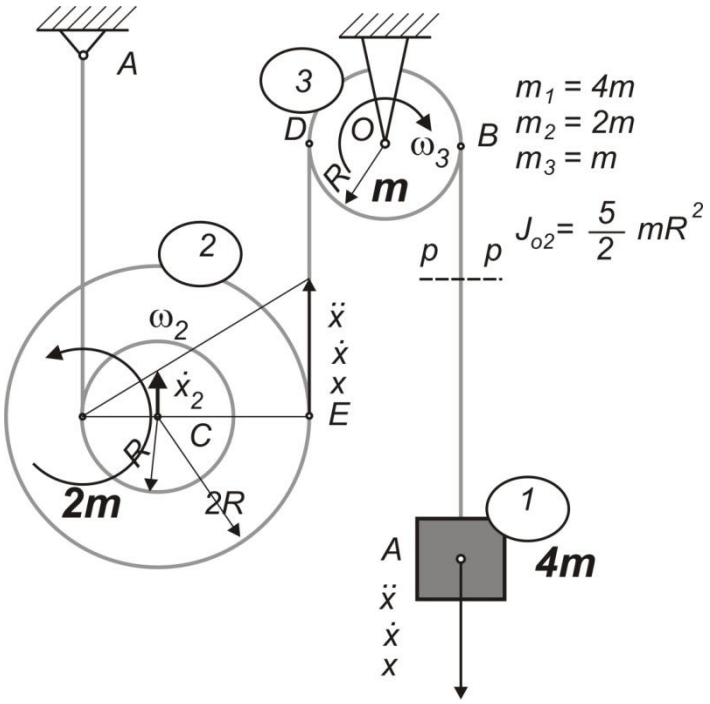


Zadatak 8.

Materijalni sistem prikazan na slici sastoji se od tereta 1 mase 4m, kalema 2 mase 2m čiji je moment inercije J_{02} kotura 3 mase m. Tela su povezana nerastegljivim užadima kao na slici. Masu užadi kojom su povezana tela zanemariti.

Odrediti: ubrzanje tereta 1, i silu u užetu u preseku p-p.

Rešenje:



Brzine tačaka

$V_A = \dot{x}$

$V_B = V_A = R\omega_3 = \dot{x}$

$\omega_3 = \frac{V_B}{R} = \frac{\dot{x}}{R} \quad \varepsilon_3 = \frac{\ddot{x}}{R}$

$V_D = R\omega_3 = V_E = 3R\omega_2$

$\omega_2 = \frac{V_D}{3R} = \frac{\dot{x}}{3R} \quad \varepsilon_2 = \frac{\ddot{x}}{3R}$

$V_C = R\omega_2 = R\frac{\dot{x}}{3R} = \frac{\dot{x}}{3}$

1) *Rešenje primenom zakona održanja energije*

Kinetička energija

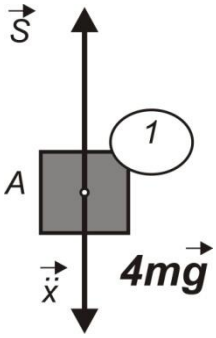
$E_K = E_k^{(1)} + E_k^{(2)} + E_k^{(3)}$

$E_k^{(1)} = E_{kTR}^{(1)} = \frac{1}{2} 4m\dot{x}^2 = 2m\dot{x}^2$

$E_k^{(2)} = E_{kTR}^{(2)} + E_{kROT}^{(2)} = \frac{1}{2} 2m \frac{\dot{x}^2}{9} + \frac{1}{2}\frac{5}{2} mR^2 \frac{\dot{x}^2}{9R^2} = \frac{9}{36} m\dot{x}^2$

$E_k^{(3)} = E_{kROT}^{(3)} = \frac{1}{2}\frac{1}{2} mR^2 \omega_3^2 = \frac{1}{4} mR^2 \frac{\dot{x}^2}{R^2} = \frac{1}{4} m\dot{x}^2$

$E_K = E_k^{(1)} + E_k^{(2)} + E_k^{(3)} = 2m\dot{x}^2 + \frac{9}{36} m\dot{x}^2 + \frac{1}{4} m\dot{x}^2 = \frac{90}{36} m\dot{x}^2 = \frac{5}{2} m\dot{x}^2$



Rad sistema

$A = A^{(1)} + A^{(2)} = 4mgx - 2mg\frac{x}{3} = \frac{10}{3} mgx$

Promena kinetičke energije jednaka je promeni rada odnosno izvršenom radu

$\frac{dE_K}{dt} = \frac{dA}{dt}$

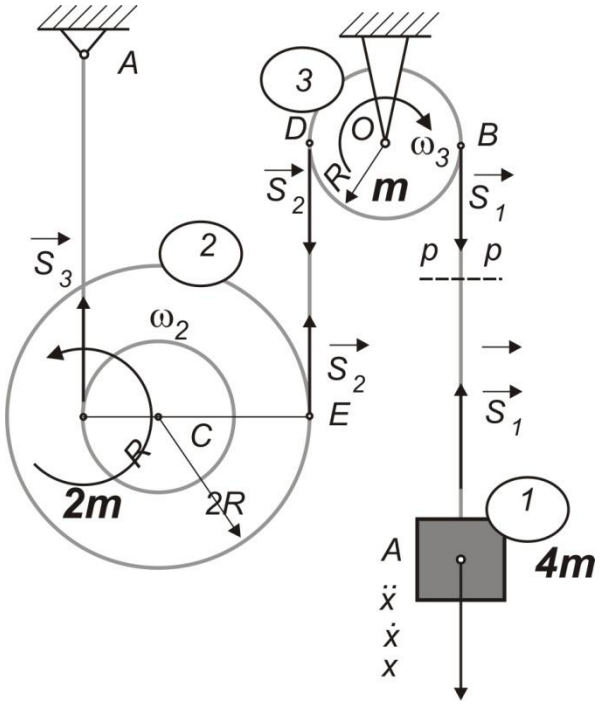
$\frac{5}{2} m2\dot{x} \cdot \ddot{x} = \frac{10}{3} mg\dot{x} \rightarrow \ddot{x} = \frac{2}{3}\, g$

Ravnoteža tereta 4m kako bi se odredila sila u užetu

$-4m\ddot{x} = S - 4mg \rightarrow S = 4m(g - \ddot{x})$

$S = 4mg\left(1 - \frac{2}{3}\right) = \frac{4}{3} mg$

2) Rešenje primenom Dalamberovog principa



1) $4m\ddot{x} = -S_1 + 4mg \rightarrow S_1$
2) $\frac{dL_O}{dt} = S_1R - S_2R \rightarrow S_2$
3) $\frac{dL_C}{dt} = S_2 2R - S_3R$
4) $2m\ddot{x}_2 = -2mg + S_2 + S_3$
5) $\varepsilon_2 = \frac{\ddot{x}}{3R}$
6) $\varepsilon_3 = \frac{\ddot{x}}{R}$

1) $S_1 = 4mg - 4m\ddot{x}$
2) $\frac{1}{2}mR^2\frac{\ddot{x}}{R} = -S_2R + S_1R$
3) $\frac{5}{2}mR^2\varepsilon_3 = S_2 2R - S_3R$ $\frac{5}{2}mR^2\frac{\ddot{x}}{3R} = \frac{5}{6}mR^2\ddot{x} = S_2 2R - S_3R$

$S_2 = S_1 - \frac{1}{2}m\ddot{x} = 4mg - \frac{9}{2}m\ddot{x}$

$S_3 = 2S_2 - \frac{5}{6}m\ddot{x} = 8mg - 9m\ddot{x} - \frac{5}{6}m\ddot{x}$

$S_3 = 8mg - \frac{59}{6}m\ddot{x}$

4) $2m\ddot{x}_1 = -2mg + S_2 + S_3 = -2mg + 4mg - \frac{9}{2}m\ddot{x} + 8mg - \frac{59}{6}m\ddot{x}$

$2m\frac{\ddot{x}}{3} + \frac{9}{2}m\ddot{x} + \frac{59}{6}m\ddot{x} = -2mg + 4mg + 8mg$

$\frac{4+27+59}{6}\, m\ddot{x} = 10\, mg$

$\ddot{x} = \frac{60}{90}\, g = \frac{2}{3}\, g$

$S_1 = 4mg - 4m\ddot{x} = 4mg - 4m \cdot \frac{2}{3}g = \frac{4}{3}mg$