normalni napon i tangencijalni napon nosača izloženog savijanju

1. $\sigma = \frac{M_f \cdot R}{I_x} = \frac{M_t}{W_x} \leq \sigma_d$
2. $\tau = \frac{M_f \cdot S_x^{'}}{I_x \cdot \xi} = \frac{M_t}{W_x} \leq \tau_d$

$$W_x = \frac{I_x}{y_{max}}$$

Zadatak 5.1.

Izračunati otporni moment pravougaonika odnosa strana b/h = ½.

$I_x = \frac{b \cdot h^3}{12}$; $I_y = \frac{h \cdot b^3}{12}$; $\frac{b}{h} = \frac{1}{2} \rightarrow h = 2b$; $b = \frac{h}{2}$

$$W_x = \frac{b \cdot h^2}{6} = \frac{I_x}{\frac{h}{2}} = \frac{\frac{b \cdot h^3}{12}}{\frac{h}{2}} = \frac{\frac{\frac{h}{2} \cdot h^3}{12}}{\frac{h}{2}} = \frac{h^3}{12}$$

$$W_y = \frac{h \cdot b^2}{6} = \frac{I_y}{\frac{b}{2}} = \frac{\frac{h \cdot b^3}{12}}{\frac{b}{2}} = \frac{\frac{h \cdot (2b)^3}{12}}{\frac{b}{2}} = \frac{\frac{8 \cdot b^3}{12}}{\frac{b}{2}} = \frac{b^3}{3}$$

Zadatak 5.2.

Izračunati otporni moment kvadrata stranice a = 3 cm.

$I_x = I_y = \frac{a^4}{12}$;

$$W_x = \frac{I_x}{\frac{a}{2}} = \frac{\frac{a^4}{12}}{\frac{a}{2}} = \frac{a^3}{6}$$

$$W_y = \frac{I_y}{\frac{a}{2}} = \frac{\frac{a^4}{12}}{\frac{a}{2}} = \frac{a^3}{6}$$

Zadatak 5.3.

Izračunati otporni moment kružnog prstena $\psi = \frac{1}{2}$;

$\psi = \frac{1}{2} = \frac{d}{D}$;

$I_x = I_y = \frac{D^4 \cdot \pi}{64}(1 - \psi^4)$;

$$W_x = \frac{I_x}{\frac{D}{2}} = \frac{\frac{D^4 \cdot \pi}{64}(1 - \psi^4)}{\frac{D}{2}} = \frac{D^3 \cdot \pi}{32}(1 - \psi^4) = \frac{D^3 \cdot \pi}{32}\left(1 - \left(\frac{1}{2}\right)^4\right) = \frac{D^3 \cdot \pi}{32}\left(1 - \frac{1}{16}\right) = \frac{15}{16} \cdot \frac{D^3 \cdot \pi}{32}$$

$$W_x = \frac{15}{16} \cdot \frac{R^3 \cdot \pi}{4} = \frac{15}{64} \cdot R^3 \cdot \pi$$

Zadatak 5.4.

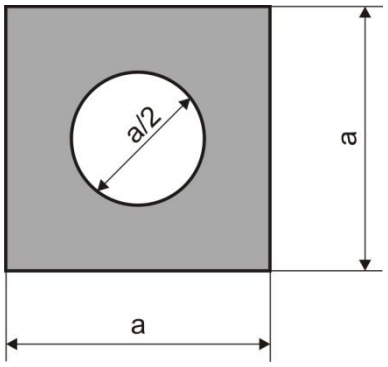
Iz kvadrata, stranice a, izvađena je centrična rupa d = ½ a.Izračunati otporni moment.

$$I_x^1 = I_y^1 = \frac{a^4}{12};$$

$$I_x^2 = I_y^2 = \frac{D^4 \cdot \pi}{64} = \frac{\left(\frac{a}{2}\right)^4 \cdot \pi}{64};$$

$$I_x = I_x^1 - I_x^2 = \frac{a^4}{12} - \frac{\left(\frac{a}{2}\right)^4 \cdot \pi}{64} = \frac{a^4}{4}\left(\frac{1}{3} - \frac{\pi}{256}\right)$$

$$W_x = \frac{I_x}{\frac{a}{2}} = \frac{\frac{a^4}{4}\left(\frac{1}{3} - \frac{\pi}{256}\right)}{\frac{a}{2}} = \frac{a^3}{2}\left(\frac{1}{3} - \frac{\pi}{256}\right)$$

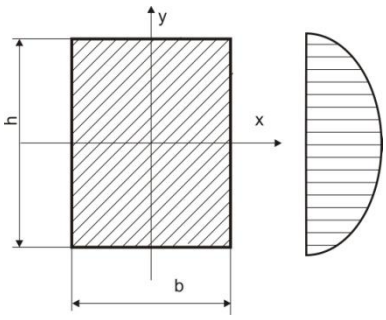


Zadatak 5.5.

Drvena pravougaona greda, preseka 6x8 cm prima tangencijalnu silu F_T = 20 kN. Koliki je najveći tangencijalni napon?

Najveći tangencijalni napon javlja se u vlaknima neutralne ose, jer je širina b=const. Obrazac za makksimalni tangelcijalni napon, obrazac 104 u knjizi Otpornost materijala, dr. Inž. D.Rašković.

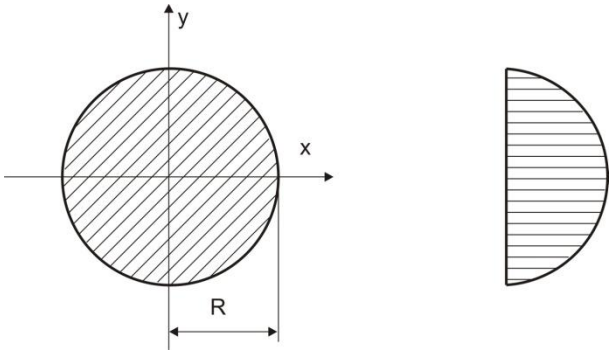
$$\tau_{max} = \frac{3}{2} \cdot \frac{F_{Tmax}}{A} = \frac{3}{2} \cdot \frac{20 \cdot 10^3}{8 \cdot 6 \cdot 10^{-4}} = 6\,250\,000\,Pa = 6.25\,MPa$$



Zadatak 5.6.

Greda kružnog preseka prima transverzalnu silu F_T = 22.25 kN. U kom odnosu stoje tangencijalni naponi u vlaknima na neutralnoj osi i osi paralelnoj njoj na udaljenju od 1/2R ako je R=50 mm?

Tangencijalni napon koji se javlja u vlaknima neutralne ose dat je izrazom za makksimalni tangelcijalni napon, obrazac 105 u knjizi Otpornost materijala, dr. Inž. D.Rašković.



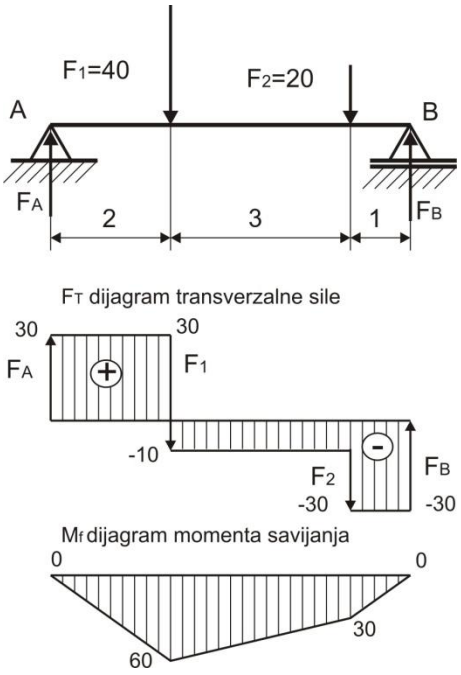
$$\tau_{max} = \frac{4}{3} \cdot \frac{F_{Tmax}}{A} = \frac{4}{3} \cdot \frac{22.25 \cdot 10^3}{\frac{5^2 \cdot \pi}{4} \cdot 10^{-4}} = 3\,828\,206\,Pa = 3.83\,MPa$$

Obrascem 105' definisan je napon u vlaknima paralelnim vlaknima neutralne ose

$$\tau = \tau_{max}\left[1-\left(\frac{y}{R}\right)^2\right] = 3\,828\,206\left[1-\left(\frac{0.5}{1}\right)^2\right] = \frac{3}{4}\cdot 3\,828\,206\,Pa = 2.87\,MPa$$

Zadatak 5.7.

Prosta greda AB raspona l = 6m, opterećena je teretima F1=40kN, F2=2kN. Dimenzionisati čelilnu gredu, kružnog poprečnog preseka, ako je dozvoljeni napon σ_d=120 MPa.



1. $\sum Z_i = 0$
2. $\sum Y_i = F_A - F_1 - F_2 + F_B = 0$
3. $\sum \mathrm{M_A} = 2\cdot \mathrm{F_1} + 5\cdot \mathrm{F_2} - 6\cdot \mathrm{F_B} = 0$

$$\mathrm{F_B} = \frac{2\cdot \mathrm{F_1}+5\cdot \mathrm{F_2}}{6} = \frac{2\cdot 40+5\cdot 20}{6} = 30\ \mathrm{kN}$$

$$F_A = F_1 + F_2 - F_B = 40 + 20 - 30 = 30\ kN$$

Dimenzionisanje poprečnog preseka nosača

$$\sigma = \frac{M_f\cdot R}{I_x} = \frac{M_t}{W_x} \le \sigma_d$$

$$W_x = \frac{d^3\cdot\pi}{32}$$

$$d = \sqrt[3]{\frac{32\,M_f}{\pi\sigma_d}} = \sqrt[3]{\frac{32\cdot 60\cdot 10^3}{\pi\cdot 120\cdot 10^6}} = 0.172\,m$$

usvojeno standardno d=0,18 m

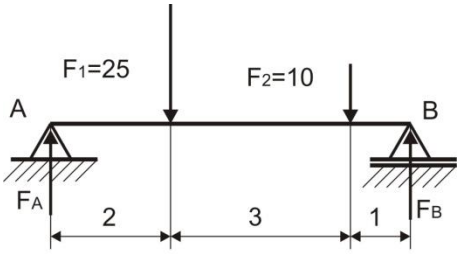
provera tangencijalnog napona

$$\tau_{max} = \frac{4}{3}\cdot\frac{F_{Tmax}}{A} = \frac{4}{2}\cdot\frac{4\cdot 30\cdot 10^3}{0.18^2\cdot\pi\cdot 10^{-4}} = 1\,571\,900\,Pa = 1.57\,MPa$$

Zadatak 5.8.

Prosta greda AB raspona l = 6m, opterećena je teretima F1=25kN, F2=10kN. Dimenzionisati čelilnu gredu, pravougaonog poprečnog preseka odnosa b/h=1/2, ako je dozvoljeni napon σ_d=120 MPa. Da li bi bio bolje iskorišćen I profil umesto pravougaonog profila?

1. $\sum Z_i = 0$
2. $\sum Y_i = F_A - F_1 - F_2 + F_B = 0$
3. $\sum \mathrm{M_A} = 2\cdot \mathrm{F_1} + 4\cdot \mathrm{F_2} - 6\cdot \mathrm{F_B} = 0$

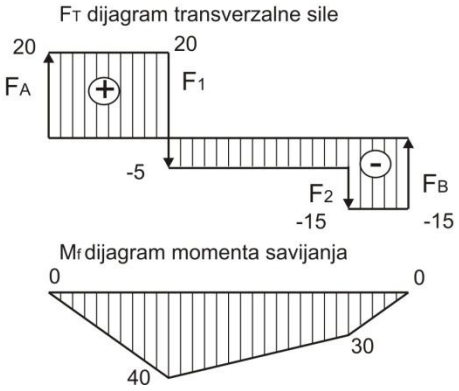


$$F_B = \frac{2\cdot F_1 + 4\cdot F_2}{6} = \frac{2\cdot 25 + 4\cdot 10}{6} = 15\ \text{kN}$$

$$F_A = F_1 + F_2 - F_B = 25 + 10 - 15 = 20\ kN$$

Maksimalni moment savijanja sa dijagrama

Mf=40 kNm



Dimenzionisanje poprečnog preseka nosača

$$\sigma = \frac{M_f \cdot R}{I_x} = \frac{M_t}{W_x} \le \sigma_d \rightarrow W_x = \frac{M_t}{\sigma_d}$$

$$W_x = \frac{M_t}{\sigma_d} = \frac{40\cdot 10^3}{120\cdot 10^6} = 3.3\cdot 10^{-4} m^3$$

$$\frac{b}{h} = \frac{1}{2} \rightarrow h = 2b\ ;\ b = \frac{h}{2}$$

$$I_x = \frac{b\cdot h^3}{12}; \quad I_y = \frac{h\cdot b^3}{12};$$

$$W_x = \frac{b\cdot h^2}{12} = \frac{I_x}{\frac{h}{2}} = \frac{\frac{b\cdot h^3}{12}}{\frac{h}{2}} = \frac{\frac{\frac{h}{2}h^3}{12}}{\frac{h}{2}} = \frac{h^3}{12}$$

$$h = \sqrt[3]{\frac{12\cdot M_t}{\sigma_d}} = \sqrt[3]{\frac{12\cdot 40\cdot 10^3}{120\cdot 10^6}} = 0.1587\ m$$

standardni pravougaoni presek bxh =0.16x0.08 m.

Težina utrošenog čelik za izradu pravougaone grede (za čelik γ=7800 kg/m^3)

$$G_{prav} = V\cdot\gamma = l\cdot b\cdot h\cdot\gamma = 6\cdot 0.16\cdot 0.08\cdot 7.8\cdot 10^3 = 599\ kg$$

Iz tablica bira se I profil čiji je Wx jednak ili veći izračunatom otpornom momentu. Izračunato je Wx =330 cm^3, što uslovljava da se izabere IPN 24 čiji je Wx=354 cm^4 koji ima težinu po jedinici dužine G'=36.2 kg/m

$$G_{INP} = G^{'}\cdot l = 6\cdot 36.2 = 217.2\ kg$$

Ušteda je u korist INP standardnog valjanog profila

$$\Delta G = G_{prav} - G_{INP} = 599 - 217.2 = 381.84\ kg$$